UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

9231 FURTHER MATHEMATICS

9231/23 Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	Use conservation of momentum for 1st collision:	$kmu_B = mu$	B1		
	Use Newton's law of restitution:	$u_B = ½u$	B1		
	Eliminate u_B to find k:	$k = 2$ **A.G.**	B1	3	
	Use conservation of momentum for 2nd collision:	$kmv_B + 6mv_C = kmu_B$	M1		
	Use Newton's law of restitution:	$v_B - v_C = -eu_B$	M1		
	Substitute and solve for v_B :	$2v_B + 6v_C = u,\ v_B - v_C = -½\, eu$			
		$v_B = (1-3e)u/8$ [$v_C = (1+e)u/8$]	M1 A1		
	Use $v_B \geq 0$ if no further collisions:	$1 - 3e \geq 0,\ e \leq ⅓$ **A.G.**	B1	5	**8**
	S.R. Taking $v_B = 0$ throughout:	$e = ⅓$	(M1 A1)	(2)	
2	Find MI of large disc about O:	$½M(3a)^2 + M(5a)^2$ [$= 59Ma^2/2$]	M1 A1		
	Find MI of small disc about O:	$½(M/9)a^2 + (M/9)a^2$ [$= Ma^2/6$]	M1 A1		
	Find MI of particle about O:	$(M/3)(8a)^2$ [$= 64Ma^2/3$]	B1		
	Sum to find MI of system about O:	**A.G.** $I = (177+1+128)\, Ma^2/6 = 51\, Ma^2$	A1	6	
	State or imply that speed is max when OP vertical		M1		
	Use energy when OP vertical (or at general point):				
		$½I\omega^2 = (5+1/9+⅓8)Mga = 70Mga/9$	M1 A1		
	Substitute for a, I and find max speed $8a\omega$:				
		$\omega = \sqrt{6{\cdot}10} = 2{\cdot}47,\ 8a\omega = 9{\cdot}9$ [ms^{-1}]	M1 A1	5	**11**
3	Moments for system about C, denoting ACB by θ:	$N_B \times BC = 2W \times 3a \cos\theta$			
		$+ W(BC - 4a \sin\theta)$ (A.E.F.)	M1 A1		
	Substitute for BC, θ:	$N_B \times 10a = 2W \times 9a/5 + W \times 34a/5$	A1		
	Simplify to give N_B:	$N_B = (26/25)\, W$ **A.G.**	A1	4	
	Find N_C by vertical resolution or moments:	$N_C = 3W - N_B = (49/25)\, W$	M1 A1		
	Find F_B (or F_C) by moments about A:	$F_B \times 24a/5 = N_B \times 32a/5$			
		$- W \times 16a/5$	M1		
		$F_B = (18/25)\, W$ *or* $0{\cdot}72W$	A1		
	Find limiting value for μ at B [or C] (A.E.F.):	$18/26$ [$= 0{\cdot}692$ *or* $18/49 = 0{\cdot}367$]	M1 A1		
	Relate F_B, F_C by e.g. horizontal resolution:	$F_C = F_B$ [$= (18/25)\, W$]	B1		
	Deduce least possible value of μ for system:	$\mu_{min} = 9/13$ *or* $0{\cdot}692$	B1	8	**12**

4	Use conservation of energy at general point:	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga(1 - \cos\theta)$	B1		
	Equate radial forces to find tension T:	$T = mg\cos\theta + mv^2/a$	B1		
	Eliminate v^2, replace u^2 by $3ag$ and simplify:	$T = mg(1 + 3\cos\theta)$ **A.G.**	M1 A1	4	
	Use energy to find speed v when PQ horizontal:	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga,\ v^2 = ga$	M1 A1		
	Use energy to find speed w when P above Q:	$\frac{1}{2}mw^2 = \frac{1}{2}mv^2 - mg(a - x)$	M1 A1		
	(note that v need not be found)	$[mw^2 = mg(2x - a)]$			
	EITHER:				
	Consider tension to find reqd. condition:	$T = mw^2/(a - x) - mg \geq$ (*or* $>$) 0	M1 A1		
	Combine to find least value of x:	$mg(3x - 2a)/(a - x) \geq 0$			
		$x \geq 2a/3,\ x_{min} = 2a/3$	M1 A1		
	OR:				
	Find x for which T becomes zero:	$mw^2/(a - x) = mg,\ x = 2a/3$	(M1 A1)		
	Show this is least possible value of x, e.g.:	$T = mg(3x - 2a)/(a - x) \geq 0$ if $x \geq 2a/3$	(M1 A1)	8	**12**
5 (i)	State or find E(X):	$E(X) = 1/0{\cdot}01$ *or* 100	B1	1	
(ii)	Integrate f(x) to find median m:	$\int_0^m f(x)\,dx = 1 - e^{-0{\cdot}01m} = \frac{1}{2}$	M1 A1		
	Solve for m:	$m = 100 \ln 2$ *or* $69{\cdot}3$	A1	3	
(iii)	Integrate f(x) to find probability:	$\int_m^{100} f(x)\,dx = \frac{1}{2} - e^{-1} = 0{\cdot}132$	M1 A1	2	**6**
6	Find pooled estimate:	$(15{\cdot}05 - 5{\cdot}5^2/5 + 36{\cdot}4 - 8^2/n)/(3+n)$	M1 A1		
	Equate to 3 and rearrange:	$45{\cdot}4 - 64/n = 9 + 3n$	M1 A1		
		$3n^2 - 36{\cdot}4n + 64 = 0$	A1		
	Solve for n:	$n = (36{\cdot}4 \pm 23{\cdot}6)/6 = 10$	M1 A1	7	**7**
7 (i)	Find probability for needing 5 throws:	$p(1 - p)^4$ with $p = 1/6$; $= 0{\cdot}0804$	M1 A1; A1	3	
(ii)	Find probability for needing < 8 throws:	$1 - (1 - p)^7 = 0{\cdot}721$	M1 A1	2	
(iii)	Relate prob. to 0·99 (allow > but not =):	$1 - (1 - p)^{n-1} \geq 0{\cdot}99$	B1		
	Find least integer n:	$(n - 1)\log 5/6 \leq \log 0{\cdot}01$	M1		
	(Allow M1 A1 even if equality used)	$n - 1 \geq 25{\cdot}3,\ n_{\min} = 27$	A1	3	**8**

8	Consider differences e.g.:	5·1 2·1 2·2 0·6 5·1 3·1 3·9 3·7	M1		
	Calculate sample mean:	$\bar{d}$ = 25·8 / 8 = 3·225	M1		
	Estimate population variance:	s^2 = (100·14 – 25·8^2/8) / 7			
	(allow biased here: 2·117 *or* 1·455^2)	[= 2·419 *or* 1·555^2]	M1		
	Find confidence interval (allow z in place of t) e.g.:	3·225 ± t √(2·419/8)	M1		
	(inconsistent use of 7 or 8 loses M1)				
	Use of correct tabular value:	$t_{7,\ 0.975}$ = 2·36[5]	A1		
	Evaluate C.I. correct to 3 s.f. (in kg):	3·225 ± 1·301 *or* [1·92, 4·53]	A1	6	
	State hypotheses:	H_0: $\mu_b - \mu_a$ = 2·5, H_1: $\mu_b - \mu_a$ > 2·5	B1		
	Calculate value of t (to 2 dp):	t = ($\bar{d}$ – 2·5)/(s/√8) = 1·32	M1 *A1		
	Compare with correct tabular t value:	$t_{7,\ 0.95}$ = 1·89[5]	*B1		
	Correct conclusion (AEF, dep *A1, *B1):	Reduction not more than 2·5	B1	5	**11**

9	(i)	Find mean values:	$\bar{x}$ = 50·1[3], $\bar{y}$ = 51·5[3]	B1		
	(ii)	Calculate gradient b' in $x - \bar{x} = b'(y - \bar{y})$:	b' = (40236 – 752 × 773/15) / (45351 – 773^2/15)	M1		
			= 1482·9 / 5515·7 = 0·268[9]	A1		
		Use regression line to estimate x at y = 56:	x = 50·13 + 0·2689 (56 – 51·53)	M1		
			[x = 36·28 + 0·2689y] = 51[·3]	A1	4	
		OR Calculate gradient b in $y - \bar{y} = b(x - \bar{x})$:				
		b = (40236 – 752 × 773/15) / (38814 – 752^2/15)		(M1)		
			= 1482·9 / 1113·7 = 1·33[15]	(A1)		
		Use regression line to estimate x at y = 56:	x = 50·13 + (56 – 51·53)/ 1·332	(M1)		
			[y = – 15·22 + 1·332x] = 53[·49]	(A1)		
	(iii)	Find correlation coefficient r:				
		r = (40236 – 752 × 773/15) / √{(38814 – 752^2/15) (45351 – 773^2/15)}		M1		
			= 1482·9 / √(1113·7 × 5515·7)			
			= 1483 / (33·37 × 74·27)			
			or 98·86 / √(74·25 × 367·7)			
			= 98·86 / (8·617 × 19·18)			
			= 0·598	*A1	2	
	(iv)	State both hypotheses:	H_0: $\rho = 0$, H_1: $\rho \neq 0$	B1		
		Use correct tabular 2-tail r value:	$r_{15,\,5\%}$ = 0·514 (to 2 dp)	*B1		
		Valid method for reaching conclusion:	Reject H_0 if $\lvert r\rvert$ > tabular value	M1		
		Correct conclusion (AEF, dep *A1, *B1):	There is a non-zero coefficient	A1	4	**11**

10 (a)	Resolve vertically at equilibrium:	$\lambda d/a = mg$ [$\lambda/a = mg/d$]	B1		
	Use Newton's Law at general point:	$m\,d^2x/dt^2 = mg - \lambda(d+x)/a$			
		[*or* $-mg + \lambda(d-x)/a$]	M1 A1		
	Simplify:	$d^2x/dt^2 = -(\lambda/ma)\,x$ *or* $-(g/d)\,x$	A1		
	S.R.: Stating this without derivation (max 3/5):		(B1)		
	Find period T using SHM with $\omega = \sqrt{(g/d)}$:	T [$= 2\pi\sqrt{(ma/\lambda)}$] $= 2\pi\sqrt{(d/g)}$ **A.G.**	B1	5	
	Use SHM formula for x with amplitude $2d$:	$x = 2d\cos(\omega t)$ [*or* sin]	M1		
	Find time t_1 to string becoming slack:	$t_1 = (1/\omega)\cos^{-1}(-1/2)$			
		or $T/4 + (1/\omega)\sin^{-1}(1/2)$	M1 A1		
	Evaluate:	**A.G.** $t_1 = (1/\omega)\,2\pi/3 = (2\pi/3)\sqrt{(d/g)}$	A1		
	Find speed v when string becomes slack:	$v = \omega\sqrt{(4d^2 - d^2)} = \omega d\sqrt{3}$ *or* $\sqrt{(3dg)}$	M1 A1		
	Find further time t_2 to instantaneous rest:	$t_2 = v/g$	B1		
	Substitute and simplify:	**A.G.** $t_2 = \sqrt{(3dg)}\,/\,g = \sqrt{3}\sqrt{(d/g)}$	M1 A1	9	**14**
(b)	Find mean and variance of sample:	$262/200 = 1{\cdot}31$ *and*			
		$(586 - 262^2/200)\,/\,200 = 1{\cdot}21[39]$	M1 A1		
	Valid comment (AEF, needs values approx correct):				
	Values close, so distn. appropriate		B1	3	
(i)	State and evaluate expression for p	**A.G.**: $p = 200\,(1{\cdot}31^2/2)e^{-1{\cdot}31} = 46{\cdot}304$	B1		
	Find q (can use $\Sigma E_i = 200$):	$q = 200\,(1{\cdot}31^3/6)e^{-1{\cdot}31} = 20{\cdot}2[19]$	B1	2	
(ii)	State (at least) null hypothesis:	H_0: Poisson fits data (A.E.F.)	B1		
	Combine last 3 cells since exp. value < 5:	O: . . . 5			
		E: . . . 8·82	*M1 A1		
	Calculate χ^2 (to 2 dp ; A1 dep *M1):	$\chi^2 = 5{\cdot}54$	M1 A1		
	Compare consistent tabular value (to 2 dp):	$\chi_{3,\,0.9}{}^2 = 6{\cdot}251$	M1 A1		
	(A1 dep *M1)	[$\chi_{4,\,0.9}{}^2 = 7{\cdot}779$, $\chi_{5,\,0.9}{}^2 = 9{\cdot}236$]			
	Valid method for reaching conclusion:	Accept H_0 if χ^2 < tabular value	M1		
	Conclusion (A.E.F., needs correct values):	5·54 < 6·25 so Poisson does fit	A1	9	**14**